647510 **BSc5MatC501x** **Seat No :________**

B.Sc. Semester - 5 (*CBCS*) Examination

Oct/Nov - 2023 (NEW COURSE)

Mathematical Analysis -1 & Abstract Algebra -1(Core)

Time: 2:30 Hours **Marks: 70**

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

Que.1(A) Answer any two out of three. (10)

1. X is metric space and $E \subset X$. Prove that E is open iff X-E is closed.
2. X is metric space. Show that intersection of finite numbers of open subsets of X is open. Explain with example that this statement is not true for infinite numbers of open subsets of X.
3. Explain cantor set and Show that $\frac{9}{10} \in C$.

Que.1(B) Answer any one out of two. (04)

1. (i) is $(A \cup B)^0 = A^0 \cup B^0$? Explain with your answer.
   (ii) Explain with example that $\overline{A \cap B} \neq \overline{A} \cap \overline{B}$.
2. Check whether set $E=(-\infty,1)$ is open and closed.

Que.2(A) Answer any two out of three. (10)

1. State and Prove necessary and sufficient condition for a bounded function f to be R-integrable.
2. if bounded functions f is R-integrable in [a,b] then show that |f| is also R-integrable and $\left|\int_a^b f dx\right| \le \int_a^b |f|\, dx$.
3. Function f is bounded in [a,b]. P and P^* be two partitions of [a,b] Such that $P \subset P^*$ Then show that $L(P,f) \le L(P^*,f)$.

Que.2(B) Answer any one out of two. (04)

1. Find $\int_1^2 x^2 \, dx$ using definition of R-integration.
2. if bounded function f on [a,b] is Monotonic then show that f is R-integrable in [a,b].

Que.3(A) Answer any two out of three. (10)

1. Using darbour's theorem find value of
$$\lim_{n\to\infty} \frac{1}{n}\left[\left(\frac{1}{n}\right)^2 + \left(\frac{2}{n}\right)^2 + \dots\dots\dots\dots + \left(\frac{n}{n}\right)^2\right]$$
2. State and prove Fundamental theorem of integration.
3. Show that set of all qube roots of 1forms a group under multiplication and find order of elements of this group.

Que.3(B) Answer any one out of two. (04)

1. if $a,b \neq 0$, $a>b$ then show that $\frac{\pi^2}{2a} < \int_0^{\pi} \frac{x}{a\cos^2\frac{x}{2} + b\sin^2\frac{x}{2}}\, dx < \frac{\pi^2}{2b}$
2. $G=R-\{-1\}$and operation * is define on G as: for $\forall\, a,b \in G$, $a*b=a+b+ab$. Check whether (G,*) is group. If G is group then find 2^{-1} in G.

Que.4(A) Answer any two out of three. (10)

1. Prove that order of permutation $f \in S_n$ is LCM of length of disjoint cycles of f.
2. Define Symmetric group and Find number of elements of Symmetric group S_n. Also write all even permutations of S_3.
3. State and prove Lagrange's theorem.

Que.4(B) Answer any one out of two. (04)

1. $G=\{(a,b)/a,b \in R$ and $a \neq 0\}$ and binary operation * define on G as: For $a,b,c,d \in R$, $(a,b)*(c,d)=(ac,bc+d)$. Show that if $H=\{(1,b)/b \in R\}$ then $H \leq G$. Where (G,*) is group.
2. $f \epsilon S_6$ where $f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 3 & 5 & 4 & 2 & 1 \end{pmatrix}$ then find f^{-1} and $O(f)$. Also Check whether f is even or odd.

Que.5(A) Answer any two out of three. (10)

1. if $(G,*) \cong (G',*')$ then show that G is commutative iff G′ is commutative. Where G and G′ be two groups.
2. Prove that for finite cyclic group, order of group is same as order of it's generator.
3. G is finite group and $a \in G$. Prove that O(a)/O(G).

Que.5(B) Answer any one out of two. (04)

1. H and K be two normal subgroups of group G. if $H \cap K=\{e\}$ then for $\forall$ $h \in H$ and $k \in K$ prove that hk=kh.
2. G is group and $H \leq G$. if index of H in G is 2 then Prove that H is normal subgroup of G.

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